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A NOTE ON THE TYPE NUMBER OF REAL HYPERSURFACES IN $P_n(C)$

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1. Introduction

Let $P_n(\boldsymbol{C})$ denote an n-dimensional complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature $4c$ and M a real hypersurface in $P_n(\boldsymbol{C})$ with the induced metric.

The problem with respect to the type number t, i.e., the rank of the second fundamental form of real hypersurfaces in $P_n(\boldsymbol{C})$ has been studied by many differential geometers ([1], [2] and [3] etc.).

The second named author [4] showed that there is a point p on M such that $t(p) \geq 2$ and M. Kimura and S. Maeda [1] gave an example of real hypersurface in $P_n(\boldsymbol{C})$ satisfying $t = 2$, which is non-complete. Y. J. Suh [3] proved that there is a point p on a complete real hypersurface M in $P_n(\boldsymbol{C})$ $(n \geq 3)$ such that $t(p) \geq 3$. According to [2], there is a point p on a complete real hypersurface M in $P_n(\boldsymbol{C})$ such that $t(p) \geq n$, but there is a mistake in deducation to lead a certain formula.

In this paper, we shall prove the following Main theorem

MAIN THEOREM. *Let M be a complete real hypersurface in $P_n(\boldsymbol{C})$ $(n \geq 4)$. Then there exists a point p on M such that $t(p) \geq 4$.*

2. Preliminaries

Let $P_n(\boldsymbol{C})$ $(n \geq 4)$ be a complex projective space with the metric of constant holomorphic sectional curvature $4c$ and M a real hypersurface in $P_n(\boldsymbol{C})$ with the induced metric. Choose a local field of orthonormal frames $e_1, \dots, e_{2n}$ in $P_n(\boldsymbol{C})$ such that $e_1, \dots, e_{2n-1}$, restricted to M, are tangent to M. We use the following convention on the range of indices unless otherwise stated: $A, B, \dots = 1, \dots, 2n$

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