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CAUCHY PROBLEMS RELATED TO DIFFERENTIAL OPERATORS WITH COEFFICIENTS OF GENERALIZED HERMITE OPERATORS

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1. Introduction

As an example, let us consider the Schrödinger equation:

$$\{D_t + D_x^2 + V(x)\}u(t,x) = 0,$$

where $D_t = (1/i)(\partial/\partial t)$, $D_x = (1/i)(\partial/\partial x)$. In the harmonic oscillator case, where the potential energy function $V(x)$ is equal to x^2, X. Feng ([1]) considered it as follows.

As is well known, the Hermite function

$$\Phi_\alpha(x) = (\alpha! 2^\alpha)^{-1/2}(-1)^\alpha \pi^{-1/4} e^{x^2/2}\left(\frac{\partial}{\partial x}\right)^\alpha e^{-x^2}$$

is an eigenfunction of the Hermite operator $H = D_x^2 + x^2$, corresponding to an eigenvalue $2\alpha + 1$, that is,

$$H\Phi_\alpha(x) = (2\alpha + 1)\Phi_\alpha(x)$$

for any $\alpha \in I_+ = \{0, 1, \ldots\}$. Moreover, $\{\Phi_\alpha \mid \alpha \in I_+\}$ is a complete orthonormal system of $L^2(R)$, and $\Phi_\alpha(x)$ belongs to $S(R)$, where $S(R)$ is the L. Schwartz space of rapidly decreasing functions in R ([2]).

Suppose $u(t,x) \in S'(R_x)$ for fixed t, where $S'(R)$ is the conjugate space of $S(R)$, and set

$$u_\alpha(t) = \langle u(t,x), \Phi_\alpha(x) \rangle.$$

Then the Cauchy problem

$$(A)\begin{cases} (D_t + H)u(t,x) = 0 & (0 \le t \le T,\ x \in R), \\ u(0,x) = \delta(x) & (x \in R) \end{cases}$$

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