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ANNIHILATOR CHARACTERIZATIONS OF DISTRIBUTIVITY, MODULARITY AND SEMIMODULARITY

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The concept of geodetic annihilators was introduced in [3]. This concept is based on the graph theoretic properties of the Hasse diagram of a finite lattice. The result of [3, Thm. 3] shows that every geodetic annihilator in a finite semimodular lattice is an intersection of prime geodetic annihilators. This and appropriate additional conditions of geodetic annihilators characterize semimodularity, modularity and distributivity in finite lattices. The graphs of these lattices are also characterized.

1. Characterizations of finite lattices

Mandelker introduced in [2] the concept of an annihilator in lattices: an annihilator $\langle a,b\rangle$ of a with respect to b is the set $\{x \mid x \wedge a \leq b\}$. The dual annihilator $\langle a,b\rangle_d$ is the set $\{y \mid y \vee a \geq b\}$. *We shall consider in this paper finite lattices only.*

Let L be a lattice. We denote the undirected Hasse diagram graph of a lattice L by G_L and call it briefly the graph of the lattice L. The distance $d(a,b)$ between two elements (vertices) a and b in a graph is the length of the shortest $a-b$ path. In graph theory, a shortest path is frequently called a geodesic. We call a set $\langle a,b\rangle_g$ of a lattice L, $a,b \in L$, a geodetic annihilator, briefly a g-annihilator, if $\langle a,b\rangle_g = \{x \mid b \text{ is on an } x-a \text{ geodesic in } G_L\}$. A set $B \subset L$ is order convex, if for any two elements $b,c \in B$ with $b \leq c$ every element x satisfying the relation $b \leq x \leq c$ belongs to B. A set B of vertices in a graph is distance convex, if for any two vertices $b,c \in B$ every vertex on any $b-c$ geodesic belongs to B. We first briefly recall some results proved in [3], which are necessary for obtaining results of this note.

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