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BRAIDING STRUCTURES OF DOUBLE CROSSPRODUCTS

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Introduction

The double crossproduct structure $X \bowtie A$ of two bialgebras (Hopf algebras) X and A is given by Majid [Mj], and Drinfel'd quantum double $D(H)$ is such a double crossproduct. Doi and Takeuchi [DT] studied the double crossproducts determined by a skew pairing. These results have some interesting application in quantum group theory. The concept of a braided bialgebra has been introduced by Larson-Towber [LTo] and Hayashi [H]. The author [C] studied the quasi-triangular structures of bicrossed coproducts. In this paper, we discuss the dual case, and study the braiding structures of double crossproducts. We also discuss the relations between the comodule categories attached to double crossproduct, and construct several (braided) monoidal functors.

1. Preliminaries

Throughout, we work over a fixed field k. Unless otherwise stated, all maps are k-linear. $\mathrm{Hom}(H,k)$ is denoted by H^*. For $f, g \in H^*$, H a bialgebra, $f * g$ is its convolution product [S]. For $\sigma \in (H \otimes H)^*$, we write $\sigma(x \otimes y) = \sigma(x, y)$, $x, y \in H$. We use the sigma notion: for $x \in H$,

$$\Delta(x) = \sum x_1 \otimes x_2, \quad (\Delta \otimes id)\Delta(x) = \sum x_1 \otimes x_2 \otimes x_3, \quad etc.$$

Let (X, A) be a pair of *matched bialgebras* (Hopf algebras), see [K], that is, X is a left A-module coalgebra via $a \rightharpoonup x$, A is a right X-module coalgebra via $a \leftharpoonup x$, such that the following conditions are satisfied:

(M1) $a \rightharpoonup (xy) = \sum (a_1 \rightharpoonup x_1)((a_2 \leftharpoonup x_2) \rightharpoonup y)$,
(M2) $a \rightharpoonup 1 = \varepsilon(a)1$,
(M3) $(ab) \leftharpoonup x = \sum (a \leftharpoonup (b_1 \rightharpoonup x_1))(b_2 \leftharpoonup x_2)$,
(M4) $1 \leftharpoonup x = \varepsilon(x)1$,
(M5) $\sum (a_1 \leftharpoonup x_1) \otimes (a_2 \rightharpoonup x_2) = \sum (a_2 \leftharpoonup x_2) \otimes (a_1 \rightharpoonup x_1)$,

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