TSUKUBA J. MATH.
Vol. 22 No. 3 (1998), 575–588

# THE INVERSE SURFACE AND THE OSSERMAN INEQUALITY*

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## 0. Introduction

In this paper, we shall work with surfaces of constant mean curvature one in hyper-bolic 3-space. We abbreviate constant mean curvature one by CMC-1. These surfaces share many properties with minimal surfaces in Euclidean 3-space. A striking result is that these surfaces have a hyperbolic analogue of Weierstrass representation formula [2]. Another important property is that the total curvature of CMC-1 surfaces is not necessarily an integral multiple of $4\pi$, and does not generally satisfy Osserman inequality [4].

Let $f: M^2 \to \boldsymbol{H}^3(-1)$ be a CMC-1 immersion. Then there exist a null holomorphic immersion $F: \tilde{M}^2 \to SL(2, C)$, such that $f = F \cdot F^*$, where $\tilde{M}^2$ is the universal cover of $M^2$. By taking the inverse of the matrix $F$, we can construct a new CMC-1 surface $f_{-1}: \tilde{M}^2 \to \boldsymbol{H}^3(-1)$, call it the inverse surface (or dual surface [5]). Although the inverse surface is defined on the universal cover $\tilde{M}^2$, its metric $ds_{-1}^2$ is well defined on $M^2$. So we have two metrics on $M^2$, and they have the same completeness [6]. Umehara and Yamada have shown that if the surface $f: M^2 \to \boldsymbol{H}^3(-1)$ is complete and of finite total curvature, then the following inequality holds

$$\frac{1}{2\pi}\int_{M^2} k_{-1}\, dA_{-1} \le \chi(M^2) - n, \tag{0.1}$$

where $n$ is the number of ends of the original CMC-1 surface, the equality holds if and only if all the ends are regular and embedded [5].

By carefully observing, we may find that the condition of finite total curvature is not necessary. Indeed we have the following theorem

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* Partially supported by NNSFC and SFECC
1991 Mathematics Subject Classification: Primary 53A10; Secondary 53C42
Key words and phrases: constant mean curvature, hyperbolic space.
Received May 28, 1997
Revised January 12, 1998