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ON SECOND SOCLES OF FINITELY COGENERATED INJECTIVE MODULES

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In [3, Theorem] Clark and Huynh proved that a right and left perfect right self-injective ring R is QF if and only if the second socle of R_R is finitely generated as a right R-module. In this note, using the technique in the proof of this theorem, we prove that if $E(T)/T$ is finitely cogenerated for every simple right R-module T, then every finitely cogenerated seminoetherian right R-module is of finite length (Theorem 5). Here, seminoetherian modules mean modules whose every nonzero submodule contains a maximal submodule. As a corollary, we obtain the theorem of Clark and Huynh (Corollary 7). Also we point out a condition for certain right perfect rings to have Morita duality (Corollary 10). In the last part of this note, we mention a dual of Theorem 5 (Theorem 13).

Throughout this note, R always denotes a ring with $J = \mathrm{Rad}(R)$. For an R-module X, $\mathrm{Soc}_k(X)$ denotes the kth socle of X for each positive integer k. For notations, definitions and familiar results concerning the ring theory we shall mainly follow [1] and [10].

First we begin with the following lemma.

LEMMA 1. *Let X and Y be right R-modules. Then*

(1) $\mathrm{Soc}_2(X \oplus Y)/\mathrm{Soc}(X \oplus Y)$ *is finitely generated if and only if* $\mathrm{Soc}_2(X)/\mathrm{Soc}(X)$ *and* $\mathrm{Soc}_2(Y)/\mathrm{Soc}(Y)$ *are finitely generated.*

(2) *If $X \leq Y$, then $\mathrm{Soc}_k(X) = \mathrm{Soc}_k(Y) \cap X$ for each positive integer k.*

PROOF. (1) This is clear from the fact that

$$\mathrm{Soc}_2(X \oplus Y)/\mathrm{Soc}(X \oplus Y) \cong \mathrm{Soc}_2(X)/\mathrm{Soc}(X) \oplus \mathrm{Soc}_2(Y)/\mathrm{Soc}(Y).$$

(2) This is a special case of [9, Proposition 3.1]. □

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