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RADIAL FUNCTIONS AND MAXIMAL ESTIMATES FOR RADIAL SOLUTIONS TO THE SCHRÖDINGER EQUATION

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1. Preliminaries and result

Let f belong to the Schwartz space $\mathscr{S}(\boldsymbol{R}^n)$ and set

$$S_t f(x) = u(x,t) = (2\pi)^{-n} \int_{\boldsymbol{R}^n} e^{ix\cdot\xi} e^{it|\xi|^a} \hat{f}(\xi)\, d\xi, \quad x \in \boldsymbol{R}^n,\ t \in \boldsymbol{R},$$

where $a > 1$. Here $\hat{f}$ denotes the Fourier transform of f, defined by

$$\hat{f} = \int_{\boldsymbol{R}^n} e^{-i\xi\cdot x} f(x)\, dx.$$

In the particular case $a = 2$, it is well known that u is the solution of the Schrödinger equation with initial data f, $i\partial u/\partial t = \Delta u$ and $\lim_{t\to 0} u(x,t) = f$ in the L^2 sense. We also introduce Sobolev spaces H_s by setting

$$H_s = \{f \in \mathscr{S}' : \|f\|_{H_s} < \infty\}, \quad s \in \boldsymbol{R},$$

where

$$\|f\|_{H_s} = \left(\int_{\boldsymbol{R}^n} (1 + |\xi|^2)^s |\hat{f}(\xi)|^2\, d\xi\right)^{1/2}.$$

We shall here consider the maximal functions

$$S^* f(\xi) = \sup_{0<t<1} |S_t f(x)|, \quad x \in \boldsymbol{R}.$$

In [3], L. Carleson proposed the question under what condition does $u(x,t) \to f$ as $t \to 0$ pointwise a.e? To answer the question it is sufficient to get an a-priori estimate of the $S^* f$.

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