

NON- c_i -SELF-DUAL QUATERNIONIC YANG-MILLS CONNECTIONS AND L_2 -GAP THEORY

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1. Introduction

In the context with the 4-dimensional Yang-Mills theory, it would be of interest to study the Yang-Mills theory on several cases which appear naturally. From this point of view, Nitta ([12]), Mamone Capria and Salamon ([8]) developed Yang-Mills theory on quaternion-Kähler manifold and gave the notion of c_1 - and c_2 -self-dual connections which reasonably corresponds to the self-dual or anti-self-dual connections on 4-dimensional manifold ([2]).

In this note, we will give two properties for c_1 - and c_2 -self-dual connections on quaternion-Kähler manifolds; (i) the existence of quaternionic Yang-Mills connections which are neither c_1 - nor c_2 -connections, and (ii) the gap phenomena for quaternionic Yang-Mills connections by L_2 -norm. These results seem natural consequence as higher dimensional analogues to 4-dimensional Yang-Mills theory.

There are remarkable results on the construction c_1 - and c_2 -self-dual connections by Kametani, Nagatomo and Nitta ([6], [9], [10], [11]). As a counter part of this result, we can consider the question whether there exist non- c_1 - and c_2 -self-dual connections on the compact quaternionic Kähler symmetric spaces, so called *Wolf spaces*. On the other hand, in 4-dimensional Yang-Mills theory, Itoh [3] found the non-self-dual Yang-Mills connections on S^4 and CP^2 . The non-self-duality of the canonical invariant G -connections on S^4 and CP^2 requires the injectivity of the isotropy homomorphisms. Namely, if the isotropy group of base space is embedded into the structure group G , then the canonical connection is not (anti-) self-dual. Employing the ideas in [3] crucially, we will give the existence of non- c_i -self-dual Yang-Mills connections in higher dimensions. Namely, we show that the canonical invariant connections on a homogeneous G -bundle with some structure group G on a Wolf space give the non- c_i -self-dual Yang-Mills connections. It is also the non- c_i -self-dual quaternionic Yang-Mills connections.

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