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TORSION INJECTIVE COVERS AND RESOLVENTS

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1. Introduction

E. Enochs began the study of injective covers in [3], characterizing when any left R-module has an injective cover. This happens if and only if R is a left noetherian ring. Torsion injective covers and torsionfree injective covers were introduced by Ahsan and Enochs in [2] and [1] respectively, in the context of the Goldie torsion theory. B. Torrecillas in [13] defined τ-torsionfree τ-injective covers and τ-injective covers for τ a hereditary torsion theory. These covers have been studied in [6] and [7].

In this paper, τ-torsion τ-injective covers and envelopes are studied for τ any torsion theory. Then we construct relative homological algebra by means of complexes with this kind of covers and envelopes. In Section 3, we find necessary and sufficient conditions for the existence of τ-torsion τ-injective covers and envelopes for any module. In the aim of the descomposition theorem of abelian groups in divisible and reduced part, we give a torsion theoretical version in terms of τ-torsion τ-injective modules as divisibles ones, Proposition 2, relating such descomposition with certain condition on the existence of τ-torsion τ-injective covers. The existence of τ-torsion τ-injective envelopes is given in Theorem 2.

In Section 4, we study the balance (see [5]) of the functor $Hom(-,-)$ relative to the class of τ-torsion τ-injective modules. When the balance is given, it is possible to introduce left derived functor of $Hom(-,-)$ by using resolvents and resolutions of τ-torsion τ-injective modules. Left and right relative global dimension of the ring R are defined and analysed.

2. Preliminaries

Throught this note R denotes a unitary ring, R-Mod the category of all left R-modules and all R-homomorphisms, and $\mathscr{C}$ a full subcategory of R-Mod

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