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THE CAUCHY PROBLEM FOR SCHRÖDINGER TYPE EQUATION WITH DEGENERACY

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§1. Introduction and Results

In this paper we consider the Cauchy problem for Schrödinger type equation

$$
(1.1)\qquad \begin{cases} \left(\partial_t + i\frac{1}{2}\sum_{j,k=1}^{n} D_j(a_{jk}(x)D_k) + \sum_{j=1}^{n} b_j(t,x)D_j + c(t,x)\right)u(t,x) \\ \quad = f(t,x) \text{ in } \mathscr{D}'((0,T)\times \boldsymbol{R}_x^n) \\ u(0,x) = u_0(x) \end{cases}
$$

where $i=\sqrt{-1}$, $D_j = -i\partial_j = -i\partial/\partial x_j$, $T>0$. We assume

$$
(A1)\qquad \begin{cases} a_{jk} \in \mathscr{B}^\infty(\boldsymbol{R}^n) \text{ real valued}, a_{jk}(x) = a_{kj}(x), 1\le j,k\le n, \\ b_j, c\in C([0,T];\mathscr{B}^\infty(\boldsymbol{R}^n)), 1\le j\le n, \\ \text{and there exists } \delta\ge 0 \text{ such that} \\ \frac{1}{2}\sum_{j,k=1}^{n} a_{jk(x)}\xi_j\xi_k \ge \delta|\xi|^2 \quad \text{for } x,\xi\in \boldsymbol{R}^n. \end{cases}
$$

Here $\mathscr{B}^\infty(\boldsymbol{R}^n) = \{f\in C^\infty(\boldsymbol{R}^n); \partial^\alpha f\in L^\infty \text{ for all } \alpha\in N^n\}$.

Put $a_2(x,\xi) = \frac{1}{2}\sum_{j,k=1}^{n} a_{jk}(x)\xi_j\xi_k, a_1(t,x,\xi) = \sum_{j=1}^{n} b_j(t,x)\xi_j$.

The purpose of this paper is to give a sufficient condition for the Cauchy problem (1.1) to be well-posed in the framework of the Sobolev spaces H^s. First we recall the related results. About the necessity, the following theorem has been shown by Ichinose [Ic.2] (resp. Hara [Ha]) in case of L^2 (resp. H^∞) well-posedness.

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