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REACHABLE SETS IN LIE GROUPS

By

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Abstract. In this paper, we deal with the right invariant control system on Lie group using norm cost, which is an alternative notion of the controllability described in [2], and local reachable sets in Lie groups of this conception was studied in [4].

1. Introduction

Let G denote a Lie group with its Lie algebra $L(G)$. We identify $L(G)$ with the set of right invariant vector fields on G. We note that $L(G)$ is linearly isomorphic to the tangent space T_eG. Since T_eG can be given the structure of a Banach space, $L(G)$ may be given the structure of a Banach space. Let Ω be a subset of $L(G)$. We consider the right invariant control system on G given by

$$(*) \qquad \dot{x}(t) = U(t)(x(t)), \quad x(0) = g,$$

where U belongs to the class $\mathscr{U}(\Omega)$ of measurable functions from $\boldsymbol{R}^+ = [0, \infty)$ into Ω which are locally bounded, and we denote the solution $x(\cdot)$ of $(*)$ by $\pi(g, \cdot, U)$, i.e., $\pi(g, 0, U) = g$ and $\pi(g, t, U) = x(t)$ for all $t \geq 0$. If there exists $U \in \mathscr{U}(\Omega)$ such that $h = \pi(g, t, U)$, then we say that h is *attainable from g at time t for the system* Ω. The set of such elements attainable from g at time t is denoted by $A(g, t, \Omega)$. We also employ the notation

$$\mathbf{A}(g, T, \Omega) = \bigcup_{0 \leq t \leq T} A(g, t, \Omega)$$

$$\mathbf{A}(g, \Omega) = \bigcup_{0 \leq t \leq \infty} A(g, t, \Omega).$$

The set $\mathbf{A}(g, \Omega)$ is called the *attainability set from g*.

Let L be a Dynkin algebra and let B be an open neighborhood of 0 which is symmetric and star-shaped in L such that for all $x, y \in B$ the Campbell-

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