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WEIERSTRASS POINTS AND RAMIFICATION LOCI ON SINGULAR PLANE CURVES

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Let X be a smooth compact Riemann surface (or a smooth projective curve) of genus g. A classical topic of study in Complex Analysis and Algebraic Geometry was the study of Weierstrass points of X. For a survey and the history of the subject up to 1986, see [EH]. For another survey containing the main definitions and results on Weierstrass points on singular Gorenstein curves, see [G]. For the case of a base field with positive characteristic, see [L]. Since Weierstrass points are "special" points on a curve, they have been very useful to study moduli problems. In particular, some subvarieties of the moduli space of smooth genus g curves are defined by the existence of suitable Weierstrass points. Several papers were devoted to the study of Weierstrass points on some interesting classes of projective curves (e.g. smooth plane curves and k-gonal curves). Our paper belong to this set of papers. We consider singular plane curves with ordinary cusps or nodes as only singularities. We believe that our paper gives a non-trivial contribution to the understanding of the existence of certain types of Weierstrass points and osculating points on these curves. In the first section we make easy extensions of [K2], Th. 1.1, to the case of singular curves. In the second section we use deformation theory to show the existence of several pairs (C, P) with C in integral nodal plane curve, $P \in C_{reg}$, such that the tangent line D of C at P has high order of contact with C at P (see Theorems 2.1 and 2.2 for precise statements). Calling X the normalization of C and seen P as a point of X, these pairs (C, P) satisfies the conditions of Proposition 1.1 below and hence P is a Weierstrass point of X. In the third section we consider the Weierstrass semigroup of the Weierstrass points obtained in this way. Here the main aim is to give a recipe to extract from the numerical calculations in [K2] as much informations as possible for the Weierstrass semigroup of the pair (X, P). The case of a total inflection point for nodal plane curves was considered in details in [CK]. Our recipe (see 3.1) gives

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