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REPRESENTATION OF q-ANALOGUE OF RATIONAL BRAUER ALGEBRAS*

By

Masashi KOSUDA

Introduction

Let q and a be indeterminates over a field K of characteristic 0, and let $K(a,q)$ denote the field of rational functions. We define the algebra $H_{m,n}(a,q)$ over $K(a,q)$ by generators and relations. (See the Definition 2.1.) If we replace the indeterminate a with q^{-r} in the definition, we have a *q-analogue of rational Brauer algebra* $H_{m,n}^{r}(q)$, which we have introduced in the previous paper with J. Murakami [8]. (In the paper [8], we called the algebra $H_{m,n}^{r}(q)$ the generalized Hecke algebra.) The algebra $H_{m,n}^{r}(q)$ is semisimple in case $r \geq m+n$, as we already observed in [8]. This observation is extended to the algebra $H_{m,n}(a,q)$. That is to say, $H_{m,n}(a,q)$ is also semisimple.

In this paper, we construct new representations of the algebras $H_{m,n}(a,q)$ and $H_{m,n}^{r}(q)$. These representations are irreducible and they are obtained from the left regular representations of $H_{m,n}(a,q)$ and $H_{m,n}^{r}(q)$ respectively.

Our previous paper was written originally to investigate the centralizer algebra of mixed tensor representations of quantum algebra $\mathcal{U}_q(gl_n(\boldsymbol{C}))$, which was q-analogue version of the work of Benkart et al. [1]. (The existence of their preliminary version of the paper [1] was informed to the author by Professor Okada.) Their original situation was as follows. Let G denote the general linear group $GL(r,\boldsymbol{C})$ of $r \times r$ invertible complex matrices and let V be the vector space on which G acts naturally. Let V^* be the dual space of V. The mixed tensor T of m copies of V and n copies of V^* is defined by $T = (\otimes^m V) \otimes (\otimes^n V^*)$. In this situation, they constructed the irreducible representations of the centralizer algebra $\mathrm{End}_G(T)$, by locating the maximal vectors in the mixed tensor T. Replacing G with $\mathcal{U}_q(gl_n(\boldsymbol{C}))$ and extending the underlying field $\mathbf{C}$ to $\mathbf{C}(q)$, we

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