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ON UNIFORM WELL-POSEDNESS OF THE ABSTRACT CAUCHY PROBLEM

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§1. Introduction

We are interested in Duhamel's principle:

$$U(t,x)=U_H(t,x;s,\Phi)+\int_s^t U_H(t,x;\tau,F(\tau,x))\,d\tau. \tag{D}$$

Here $U(t,x)$ denotes a solution of the inhomogeneous Cauchy problem $[CP]_s$ $(0\le s<T)$ for the system of linear differential equations:

$$\begin{cases} L(t,x;\partial_t,\partial_x)U(t,x)=F(t,x),t\in[s,T],x\in\boldsymbol{R}^n, \\ U(s,x)=\Phi(x) \end{cases}$$

and $U_H(t,x;s,\Phi)$ stands for a solution of the homogeneous Cauchy problem $[HCP]_s$ $(0\le s<T)$ for the system of linear differential equations:

$$\begin{cases} L(t,x;\partial_t,\partial_x)U(t,x)=0,t\in[s,T],x\in\boldsymbol{R}^n, \\ U(s,x)=\Phi(x), \end{cases}$$

where $L(t,x;\partial_t,\partial_x)=\partial_t I-A(t,x;\partial_x)$, I is the identity matrix of order n and $A(t,x;\partial_x)$ is a square matrix of order n.

When L is an ordinary differential operator, that is, $A(t,x;\partial_x)$ depends only on t, write $L=\partial_t I-A(t)$, the formula (D) becomes the constant variation formula of Lagrange

$$U(t)=\varphi(t)\varphi(s)^{-1}\Phi+\int_s^t \varphi(t)\varphi(\tau)^{-1}F(\tau)\,d\tau,$$

where $\varphi(t)$ is the fundamental matrix to $[HCP]_s$. In particular, if $A(t)$ is in-

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