TSUKUBA J. MATH.
Vol. 21 No. 3 (1997), 567–593

MORSE THEORY AND NON-MINIMAL SOLUTIONS TO THE YANG-MILLS EQUATIONS

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Abstract. By generalizing a method of Taubes, we use Morse theory to find the higher critical points of Yang-Mills functional.

Topology and Analysis name mathematical subjects with robust interaction, often along the basic theme: Study relationship between the critical points of functional and the topology of function space. We consider in this article a vector type non compact variational problem—the Yang-Mills equations, and we raise the question of proving the existence of "true critical points" for this functional, in a framework where the Palais-Smale condition does not hold.

This article should be considered as sequel to [27], where the most of the notations and the terminology were introduced. The reader may find that the exposition in [13], [22], [24], [26], and [27] are useful introductions to Morse theory for the Yang-Mills equations. The main purpose of this paper is that it clearly explains the background and motivation, and gives a method for finding the non-minimal solutions to the Yang-Mills equations on a compact oriented 4-manifold.

In our main result, we suppose that there is a known isolated non-minimal Yang-mills field ("isolated means that the Hessian of Yang-Mills functional is non-degenerate). We then use the min-max method to produce infinitely many other non-minimal Yang-Mills fields.

THEOREM. *Let M be a compact oriented Riemannian 4-manifold. Let A_0 be an isolated non-minimal Yang-Mills connection on M with the structure group $SU(2)$ such that $d|P_{\pm}F_{A_0}| = 0$ along a simple closed geodesic and $|P_{\pm}F_{A_0}| > 0$ on this geodesic. Then there is a constant $K > 0$ such that for any positive even integer $k > K$, there exists an irreducible non-minimal Yang-Mills $SU(2)$ connection with the same degree as A_0.*

Received November 1, 1995.
Revised March 29, 1996.