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KRONECKER FUNCTION RINGS OF SEMISTAR-OPERATIONS

By

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1. Introduction

Let D be a commutative integral domain with quotient field K. Let $F(D)$ denote the set of non-zero fractional ideals of D in the sense of [K], i.e., non-zero R-submodules of K and let $F'(D)$ denote the subset of $F(D)$ consisting of all members A of $F(D)$ such that there exists some $0 \neq d \in D$ with $dA \subset D$. Let $f(D)$ be the set of finitely generated members of $F(D)$. Then $f(D) \subset F'(D) \subset F(D)$.

A mapping $A \rightarrow A^*$ of $F'(D)$ into $F'(D)$ is called a *star-operation* on D if the following conditions hold for all $a \in K - \{0\}$ and $A, B \in F'(D)$:

(1) $(a)^* = (a), (aA)^* = aA^*$;
(2) $A \subset A^*$; if $A \subset B$, then $A^* \subset B^*$; and
(3) $(A^*)^* = A^*$.

A fractional ideal $A \in F'(D)$ is called a $*$*-ideal* if $A = A^*$. We denote the set of all $*$-ideals of D by $F_*(D)$. A star-operation $*$ on D is said to be *of finite character* if $A^* = \bigcup\{J^* | J \in f(D) \text{ with } J \subset A\}$ for all $A \in F'(D)$. It is well known that if $*$ is a star-operation on D, then the mapping $A \rightarrow A^{*_f}$ of $F'(D)$ into $F'(D)$ given by $A^{*_f} = \bigcup\{J^* | J \in f(D) \text{ with } J \subset A\}$ is a finite character star-operation on D. Clearly we have $A^* = A^{*_f}$ for all $A \in f(D)$ and all star-operations $*$ on D.

The mapping on $F'(D)$ defined by $A \rightarrow A_v = (A^{-1})^{-1}$ is a star-operation on D and is called the *v-operation* on D, where $A^{-1} = \{x \in K | xA \subset D\}$. The *t-operation* on D is given by $A \rightarrow A_t = \bigcup\{J_v | J \in f(D) \text{ with } J \subset A\}$, that is, $t = v_f$. The reader can refer to [G, Sections 32 and 34] for the basic properties of star-operations and the v-operation.

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