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C^*-EMBEDDING AND C-EMBEDDING ON PRODUCT SPACES

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1. Introduction

Throughout this paper by a space we mean a topological space. Let X be a space and A its subspace. Then A is said to be *C^*-embedded* (resp. *C-embedded*) in X if every bounded real-valued (resp. real-valued) continuous function on A can be extended to a continuous function over X. For an infinite cardinal number γ, A is said to be *P^{γ}-embedded* in X if for every locally finite cozero-set cover $\mathscr{U}$ of A with Card $\mathscr{U} \leq \gamma$ there exists a locally finite cozero-set cover $\mathscr{V}$ of X such that $\mathscr{V} \cap A$ $(= \{V \cap A | V \in \mathscr{V}\}) < (=\text{refines})\mathscr{U}$; A is *P-embedded* in X if A is P^{γ}-embedded in X for every γ. P^{γ}- and P-embeddings were originally introduced by Shapiro [16]. For the case $\gamma = \aleph_0$ it is known that $P^{\aleph_0}$-embedding coincides with C-embedding. And a well-known fact is that collectionwise normal spaces are those spaces in which every closed subset is P-embedded. For basic facts of these embeddings the reader is referred to Alò and Shapiro [1] and Hoshina [3].

As for normality of product spaces we have known the following results due to Morita [4] and Rudin and Starbird [15], respectively, that a Hausdorff space X is γ-paracompact normal iff $X \times Y$ is normal for any compact Hausdorff space Y of weight $w(Y) \leq \gamma$, and that for a normal space X and a non-discrete metric space $Y, X \times Y$ is normal iff $X \times Y$ is countably paracompact. Being motivated by the first result Morita and Hoshina [7] and Przymusiński [12] independently proved that for a compact Hausdorff space Y with $w(Y) = \gamma$, A is P^{γ}-embedded in X iff $A \times Y$ is C^*-embedded in $X \times Y$. On the other hand, corresponding to the second result above, the following problem was posed in Przymusiński [13] (see Hoshina [3]) but still remains open: for a non-discrete metric space Y is it true that $A \times Y$ is C^*-embedded in $X \times Y$ iff $A \times Y$ is C-embedded in $X \times Y$? Recently Ohta [10] proved this equivalence when $Y = \kappa^{\omega}$,

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