TSUKUBA J. MATH.
Vol. 21 No. 2 (1997), 505–514

ON THE EXISTENCE AND NONEXISTENCE OF GLOBAL SOLUTIONS OF SEMILINEAR PARABOLIC EQUATIONS WITH SLOWLY DECAYING INITIAL DATA

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1. Introduction

In this paper, we study the Cauchy problem

$$\text{(P)}\qquad \begin{cases} u_t = \Delta u + |x|^{\sigma} u^p & \text{in } D \times (0, T), \\ u(x,t) = 0 & \text{on } \partial D \times (0, T), \\ u(x,0) = a(x) \geq 0 & \text{in } D, \end{cases}$$

where $\sigma \geq 0, p > 1$.

The domain D is a cone in $\boldsymbol{R}^N$, such as

$$D = \{x \in \boldsymbol{R}^N \backslash 0; x/|x| \in \Omega\},$$

where $\Omega \subset \boldsymbol{S}^{N-1}$ is an open connected subset with smooth boundary. The nonnegative initial condition $a(x)$ is continuous and $\langle x \rangle^{\sigma/(p-1)} a(x)$ is bounded in $\bar{D}$ and $a = 0$ on ∂D ($\langle x \rangle = \sqrt{1+|x|^2}$).

Let ω_1 be the smallest Dirichlet eigenvalue for the Laplace-Beltrami operator on Ω, and γ_+ be the positive root of $\gamma(\gamma + N - 2) = \omega_1$. The following results are well known by the papers of Levine and Meier [3], [4] and Hamada [2].

(I) If $1 < p \leq 1 + (2+\sigma)/(N+\gamma_+)$, there is no nontrivial nonnegative global solution for any initial data.

(II) If $p = 1 + (2+\sigma)/(N+\gamma_+)$, there is no nontrivial nonnegative global solution for any initial data.

(III) If $p > 1 + (2+\sigma)/(N+\gamma_+)$, then there exist nontrival nonnegative global solutions for sufficiently small initial data.

Received September 21, 1995