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GLOBAL SOLVABILITY FOR THE GENERALIZED DEGENERATE KIRCHHOFF EQUATION WITH REAL-ANALYTIC DATA IN R^n

By

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1. Introduction

Kirchhoff equation was proposed by Kirchhoff in 1883 to describe the transversal oscillations of a stretched string and it is expressed as follows

$$\partial_t^2 u(t,x) - \left(\varepsilon^2 + \frac{1}{2l}\int_0^l |\partial_x u(t,x)|^2 dx\right)\partial_x^2 u(t,x) = 0, \tag{1.1}$$

where $t > 0$, $l > 0$, $\varepsilon > 0$ and $x \in [0,l]$. In 1940 S. Bernstein [B] proved the global solvability for analytic initial data and local solvability for C^m*-class* initial data to the following initial boundary value problem:

$$\begin{cases} \partial_t^2 u(t,x) - \left(a + b\int_0^{2\pi} |\partial_x u(t,x)|^2 dx\right)\partial_x^2 u(t,x) = 0 \quad (t > 0, x \in [0,2\pi]), \\ u(t,x) = 0 \quad (t \geq 0, x = 0, 2\pi), \\ u(0,x) = u_0(x), \quad \partial_t u(0,x) = u_1(x), \end{cases} \tag{1.2}$$

where $a > 0$ and $b > 0$. In 1971, T. Nishida [Nd] proved Bernstein's result in case of $a = 0$. Equation (1.2) can be regarded as the following more generalized equation:

$$\begin{cases} \partial_t^2 u(t,x) - M\left(\int_\Omega |\nabla_x u(t,x)|^2 dx\right)\Delta_x u(t,x) = 0 \quad (t > 0, x \in \Omega), \\ u(0,x) = u_0(x),\ \partial_t u(0,x) = u_1(x), \quad x \in \Omega \subset \boldsymbol{R}^n, \end{cases} \tag{1.3}$$

with boundary condition

$$u(t,x) = \varphi \quad \text{on } [0,\infty) \times \partial\Omega. \tag{1.4}$$

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