TSUKUBA J. MATH.
Vol. 21 No. 2 (1997), 443–448

# REMARKS ON SPACES WITH SPECIAL TYPE OF $k$-NETWORKS

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**Abstract:** We negatively answer the following questions posed by Y. Ikeda and Y. Tanaka. (1) Does every closed image of a space $X$ with a star-countable $k$-network have a star-countable $k$-network, or a point-countable $k$-network? (2) Is every space $X$ with a locally countable $k$-network a $\sigma$-space, or a space in which every closed subset is a $G_\sigma$-set?

## 1. Introduction

All spaces we consider here are completely regular Hausdorff and all maps are continuous and onto. A collection of subsets of a space is said to be *star-countable* (resp. *point-countable*) if each element (resp. single point) meets only countably many members. Obviously a star-countable collection is point-countable. A collection $\mathscr{P}$ of subsets of a space $X$ is called a *k-network* if whenever $K$ is a compact subset of an open set $U$, there exists a finite subset $\mathscr{P}'$ of $\mathscr{P}$ such that $K \subset \bigcup\mathscr{P}' \subset U$. If we replace "compact" by "single point", then $\mathscr{P}$ is called a *network*. A space with a $\sigma$-locally finite network is called a *$\sigma$-space*.

Concerning spaces with special type of $k$-networks, Y. Ikeda and Y. Tanaka posed the following questions in [7], see also [10] and [12].

QUESTIONS. (1) Does every closed image of a space $X$ with a star-countable $k$-network have a star-countable $k$-network, or a point-countable $k$-network?

(2) Is every space $X$ with a locally countable $k$-network a $\sigma$-space, or a space in which every closed subset is a $G_\sigma$-set?

The question (1) has a positive answer under some conditions.

Received July 31, 1995.
Revised November 20, 1995.