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SPECTRA OF THE LAPLACIAN ON THE CAYLEY PROJECTIVE PLANE

By

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Dedicated to Professor Hideki Ozeki on his sixtieth birthday

Introduction

Let $M = G/K$ be a compact homogeneous space of a compact semi-simple Lie group G. Let V be a complex homogeneous vector bundle on M. The group G acts naturally on the space of sections $\Gamma(V)$ of V. By a theorem of Peter and Weyl, $\Gamma(V)$ is a unitary direct sum of finite dimensional representations of G. It is an important problem to decompose $\Gamma(V)$ into irreducible G-modules. By the Frobenius reciprocity theorem, the problem is divided into two parts:

1. How does an irreducible G-module decompose as a K-module (branching law)?
2. How does the fiber V_0 decompose as a K-module?

In spite of its importance there are not so many pairs (G, K) of which the branching law is investigated. For instance, see the list in Strese [7]. The branching law of the compact symmetric pair of rank one are fully explained except the case $(F_4, Spin(9))$. On the branching law of the pair $(F_4, Spin(9))$, we have a result of Lepowsky [5]. But his result is not sufficient to decompose the space of sections $\Gamma(V)$.

A section of $\bigwedge^p(T^*M^C)$ is a (complex) p-form on M. Since the Laplacian on M acting on p-forms commutes with the action of G, Δ is a scalar operator on each irreducible component of $\bigwedge^p(T^*M^C)$ and the eigenvalue is calculated by Freudenthal's formula [3]. By this program, the spectra of p-forms on spheres and complex projective spaces are calculated by Ikeda and Taniguchi [3], and the spectra of quaternion projective spaces and real Grassmann manifolds of 2-planes are calculated by Strese [8] and Tsukamoto [9]. The

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