

SEPARABLE FUNCTORS IN COALGEBRAS. APPLICATIONS

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Introduction

The notion of separable functor was introduced in [5], where some applications in the framework of group-graded rings were done. This notion fits satisfactorily to the classical notion of separable algebra over a commutative ring. The concept of coseparable coalgebra over a field appears in [1] to prove a result of Sullivan [7]. A more complete study of the separability of coalgebras was performed in [2]. In this last paper, an analysis of the relationship between coseparability and the cohomology theory for coalgebras is developed.

Our aim is to study the separability, in the sense of [5], of some canonical functors stemming from a morphism of coalgebras.

In Section 1 we fix some notation and we prove a preliminary characterization of the bicomodules.

The Section 2 contains the theoretical body of the paper. For a morphism of coalgebras $\varphi : C \rightarrow D$, we characterize the separability of the corestriction functor $(-)_\varphi$ (Theorem 2.4) and of the coinduction functor $(-)^{\varphi}$ (Theorem 2.7). The reader can find the definitions of these functors in Section 1. For the particular case of the coalgebra morphism $\varepsilon : C \rightarrow k$ given by the counit of the k -coalgebra C , the separability of the corestriction functor gives precisely the notion of coseparable coalgebra. We finish the section with Theorem 2.9, that entails that a coseparable coalgebra need not to be necessarily of finite dimension (Theorem 3.4).

Section 3 is devoted to study the relationship between coseparability and co-semisimplicity for coalgebras. As a consequence, we obtain that a k -coalgebra

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