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ON GLOBAL HYPOELLIPTICITY ON THE TORUS

By

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Summary: We use Fourier series and continued fractions to study the property of regularity of the global solutions of certain partial (or pseudo-) differential equations on the torus.

1. Introduction

Our main purpose in this paper is to study global hypoellipticity for a class of pseudo-differential operators on the n-Torus, T^n, $n \geq 2$, of the form

$$P = p(D_1^2) + e^{imx_1} + ae^{-imx_1},$$

where $a = \pm 1$, $m \in N$, $D_1 = (1/i)(\partial/\partial x_1)$ and p is a classical symbol satisfying the additional conditions:

$$p(0) = 0; \quad |p(1)| \geq 1; \quad |p(t^2)| > 2, \quad t \in N, \quad t \geq 2. \tag{1}$$

We recall that an operator P is said to be **globally hypoelliptic** (GH) on T^n if the properties $u \in \mathcal{D}'(T^n)$ and $Pu \in C^\infty(T^n)$ imply $u \in C^\infty(T^n)$.

Under hypothesis (1), we present a **necessary and sufficient** condition for the operators in (1) to be (GH). Our examples show, in particular, that in the case when $p(t) = \lambda t^2, 1 < \lambda < 2$, the situation $m > 1$ is different from the case $m = 1$, (see [5]); namely, when $m > 1$, the operator may fail to be (GH).

Other related works dealing with global hypoellipticity are [6], [7], [1]. In [6] the operators $D_1^2 + 2\cos x_1 - \lambda, \lambda \in C$, are considered; in [7] this result is extended to cover more general operators with the same perturbation of order zero. In [1], the effect of perturbations by terms of order zero is considered only in the case of constant coefficients. Further related recent works are [2], [3].

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