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REPRESENTATIONS OF A LINK GROUP

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In this paper we shall consider the link L illustrated in the Figure 1, and construct representations of the link group of L.

The link group $\pi(L)$ has the following presentations:

$$\pi(L) \simeq \langle x_0, x_1, x_2, x_3, x_4, x_5 | [x_i, x_{i-1}x_{i+1}^{-1}] = 1,\ i = 0, 1, \ldots 5 \ (\text{mod. } 6)\rangle$$

$$\simeq \langle x_0, x_1, x_2, x_3, x_4, x_5, y_0, y_1, y_2, y_3, y_4, y_5 |$$

$$[x_i, y_i] = 1,\ x_{i-1}x_{i+1}^{-1}y_i^{-1} = 1,\ i = 0, 1, \ldots, 5 \ (\text{mod. } 6)\rangle,$$

where $[x, y] = xyx^{-1}y^{-1}$. Note that, in the last presentation, it holds that $y_0y_2y_4 = 1$ and $y_1y_3y_5 = 1$.

Now, representations of $\pi(L)$ to $\mathrm{PSL}(2, \boldsymbol{C})$ can be constructed using the following theorem. We set $c(z) = (z + 1/z)/2$ and $s(z) = (z - 1/z)/2$.

THEOREM. Let λ_0, λ_2, λ_4, μ_0, μ_2, μ_4 be complex numbers not equal to $0, \pm 1$. Suppose that we can take complex numbers λ_1, λ_3, λ_5, μ_1, μ_3, μ_5 not equal to $0, \pm 1$, satisfying the following conditions (i) and (ii): for $i = 1, 3, 5$ (mod. 6)

$$\text{(i)}\quad c(\lambda_i) = c(\mu_{i-1})c(\mu_{i+1}) + \{c(\lambda_{i-1})c(\lambda_{i+1}) - c(\lambda_{i+3})\}s(\mu_{i-1})s(\mu_{i+1})/\{s(\lambda_{i-1})s(\lambda_{i+1})\},$$

$$\text{(ii)}\quad s(\lambda_i)c(\mu_i)/s(\mu_i) = -c(\lambda_{i-1})c(\mu_{i+1})s(\mu_{i-1})/s(\lambda_{i-1}) - c(\mu_{i-1})c(\mu_{i+1})s(\mu_{i+1})/s(\lambda_{i+1}) - c(\mu_{i+3})s(\lambda_{i+3})s(\mu_{i-1})s(\mu_{i+1})/\{s(\mu_{i+3})s(\mu_{i-1})s(\mu_{i+1})\}.$$

Then, we can take $A_i \in \mathrm{SL}(2, \boldsymbol{C})$ $(i = 0, 1, \ldots, 5 \ (\text{mod.} 6))$ to construct a non-abelian representation of $\pi(L)$ to $\mathrm{PSL}(2, \boldsymbol{C})$ by corresponding

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