TSUKUBA J. MATH.
Vol. 21 No. 2 (1997), 263–283

GLOBAL ATTRACTORS FOR PARABOLIC P.D.E.'S IN HÖLDER SPACES

By

Jan W. CHOLEWA and Tomasz DLOTKO

Abstract. The $2m$-th order semilinear parabolic p.d.e. is shown to generate a strongly continuous semigroup $T(t)$ of classical solutions on a subspace X^μ of the space of Hölder functions $C^{2m+\mu}(cl\Omega)$. Based on the smoothing action of parabolic equations and the *dissipativity* of $T(t)$ in fractional power spaces the existence of a global attractor for $T(t)$ in the topology of X^μ is then justified. Some applications, e.g. to the famous Cahn-Hilliard equation, are finally discussed.

1. Introduction

1.1. Motivation.

It is well known that the *global attractor* contains much of the relevant information about dynamics of the system [7]. One of many advantages it offers, resulting from its strong stability properties, is the possibility of controlling the behaviour of the system after a long time. When semiflows generated by p.d.e.'s are considered there are, however, a number of possible choices of phase spaces and consequently properties of the attractor may often be displayed in the frame of various topologies. It might then be important in applications to get an attractor on a possibly 'small' space (i.e. in a strong topology) or, when it is known in a 'larger' space, where its existence is easier to prove, to show that it is the same set in a 'smaller' one. For partial differential equations, this impose the need to study semigroups of very smooth solutions which usually leads to additional technical difficulties. Nevertheless, more regularity of the semigroup

1991 *Mathematics Subject Classification.* 35K25, 35B40.
Key words and phrases. Parabolic equations, smooth global solutions, dissipative semigroups in Hölder spaces, smooth global attractors.
Supported by the State Committee for Scientific Research Grant No. 2 P301 032 05.
Received December 19, 1994.
Revised February 24, 1997.