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ON ALMOST KÄHLER MANIFOLDS OF CONSTANT CURVATURE

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§1. Introduction

An almost Hermitian manifold $M=(M, J, g)$ is called an almost Kähler manifold if the corresponding Kähler form is closed (or equivalently $\mathfrak{S}_{X,Y,Z}g((\nabla_X J)Y, Z)=0$ for $X, Y, Z \in \mathfrak{X}(M)$, where $\mathfrak{S}$ and $\mathfrak{X}(M)$ denotes the cyclic sum and the Lie algebra of all differentiable vector fields on M respectively). A Kähler manifold, which is defined by $\nabla J=0$, is necessarily an almost Kähler manifold. It is well-known that an almost Kähler manifold with integrable almost complex structure is a Kähler manifold. A non-Kähler almost Kähler manifold is called a strictly almost Kähler manifold. Concerning the integrability of almost Kähler manifolds, the following conjecture by S. I. Goldberg is known ([2]):

CONJECTURE. *A compact almost Kähler Einstein manifold is a Kähler manifold.*

K. Sekigawa proved the above conjecture is true for the case where the scalar curvature is nonnegative ([7]). However, the above conjecture is still open in the case where the scalar curvature is negative.

Concerning the above conjecture, Z. Olszak proved that, in dimensions ≥ 8, an almost Kähler manifold of constant curvature is a flat Kähler manifold ([6]). In dimension 4, D. E. Blair claimed that the same assertion is valid by making use of quaternionic analysis. However, there is a gap in the final step of his proof. The statement "each $a_i=0$" is not correct ([1], p. 1038). Recently, K. Sekigawa and the author proved that a $2n(\geq 4)$-dimensional complete almost Kähler manifold of constant sectional curvature is a flat Kähler manifold ([5]). The proof in [5] is essentially dependent on the completeness.

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