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SPACELIKE SUBMANIFOLDS WITH PARALLEL MEAN CURVATURE IN PSEUDO-RIEMANNIAN SPACE FORMS

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1. Introduction

In the last years, several authors have studied spacelike hypersurfaces with constant mean curvature in Lorentzian spaces of constant curvature, see for instance [2], [10], [11]. When the codimension of the spacelike submanifold is greater that one, the natural generalization, that is, the case of parallel mean curvature vector in the normal bundle, has been dealt in [1], [3], [5]. From a technical point of view, the closest case to that of spacelike hypersurfaces is when the codimension is equal to the index of the ambient space, so that, when the normal bundle is negative definite [1], [5], [9]. Under this assumption, it has been mainly used as a tool classical Simons' formula for the Laplacian of the length of the second fundamental form. However, this technique does not seen to be useful when the normal bundle is not definite. In [3], a different method has been introduced to study compact spacelike submanifolds with parallel mean curvature vector in de Sitter spaces, with non-definite normal bundle. As any index for the normal bundle was allowed, an assumption on the Ricci curvature (automatically satisfied in the definite case) was shown to be necessary. In this paper we will study compact spacelike submanifolds with (non-zero) parallel mean curvature vector in a pseudo-Riemannian space form with Lorentzian normal bundle (of signature $(1,p)$). We use the same approach as in [3]; however no assumption on the curvature of the submaifold is now made, and the family of ambient spaces is extended in order to consider flat and negatively curved pseudo-Riemannian space forms. Our study was first motivated by the following easy fact: consider a totally umbilical and non-totally geodesic hypersurface M^n of a Riemannian space form of sectional curvature $c \in \boldsymbol{R}$, $N^{n+1}(c)$. Embedding $N^{n+1}(c)$ as a totally geodesic submanifold in an

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