TSUKUBA J. MATH.
Vol. 21 No. 1 (1997), 153-168

STRUCTURE THEOREMS OF THE SCALAR CURVATURE EQUATION ON SUBDOMAINS OF A COMPACT RIEMANNIAN MANIFOLD

By

Shin KATO

1. Introduction

Let (M, g) be a Riemannian manifold with $\dim M = n \geq 3$, Δ_g the Laplacian of g, S_g the scalar curvature of g and L_g the conformal Laplacian of g, i.e. $L_g := -a_n\Delta_g + S_g$ with $a_n = 4(n-1)/(n-2)$. Let u be a positive smooth function on M, and define a conformal metric by $\tilde{g} := u^{4/(n-2)}g$. Then its scalar curvature is given by $S_{\tilde{g}} = u^{-q}L_g u$, where $q = (n+2)/(n-2) = 4/(n-2)+1$. Hence, a smooth function f on M can be realized as the scalar curvature of some metric which is pointwise conformal to g if and only if there is a smooth solution u of the equation

$$\begin{cases} L_g u = f u^q \\ u > 0 \end{cases} \quad \text{on } M.$$

Throughout this paper, we refer to this equation as "the equation (f, M)".

Now, we are interested in the structure of the moduli space of (complete) conformal metrics on M with scalar curvature f. In this work, we study the equation (f, M) in the case when (M, g) is a subdomain of a compact Riemannian manifold $(\overline{M}, \bar{g})$. More precisely, we consider mainly the case when $\lambda_1(L_{\bar{g}}) > 0$, (M, g) is the complement $\overline{M}\backslash\Sigma$ of a compact submanifold Σ, and f is nonpositive.

Under this assumption, Mazzeo [12] proved that, when $d = \dim\Sigma \leq (n-2)/2$ and $f \equiv 0$ on M, "the full solution space of scalar flat complete conformal metrics on M is parametrized by the space of strictly positive measures on Σ." This fact means that Σ is the Martin boundary of the Laplacian with respect to a scalar flat complete conformal metric on M.

When f has a compact support, any conformal metric $u^{q-1}g$ on M with scalar curvature f is bounded above by some scalar flat conformal metric $\varphi^{q-1}g$

Received February 6, 1995.
Revised August 30, 1995.