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REDUCTION PROPERTY AND DIMENSIONAL ORDER PROPERTY

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1. Introduction

Let T be a theory with a relational language L including a unary predicate P. Let M be a model of T and N the L^--structure $P^M = \{a \in |M| : M \models P(a)\}$ where $L^- \subset L - \{P\}$. The following question seems to be natural:

QUESTION. Which properties of $T^- = Th(N)$ are also possessed by T (under certain conditions)?

There are a few papers treating the question. In [HP] Hodges and Pillay have shown that if T is minimal over P (definition 2.3) and every automorphism of N can be extended to an automorphism of M (they call M is a symmetric extension of N), then N is $\aleph_0$-categorical iff M is $\aleph_0$-categorical. In [KT] Kikyo and Tsuboi defined the $\varnothing$-reduction property, the reduction property, the strong reduction property, and the uniform reduction property. These reduction properties are model theoretical rephrasing of symmetry. They have shown that if T is minimal over P and has the uniform reduction property (i.e., for each L-formula $\varphi(\bar{x}\bar{y})$, there is an L^--formula $\psi(\bar{x}\bar{z})$ such that $(\forall\bar{y})(\exists\bar{z} \in P)(\forall\bar{x} \in P)[\varphi(\bar{x}\bar{y}) \leftrightarrow \psi^P(\bar{x}\bar{z})]$ holds), then T^- is λ-stable iff T is λ-stable and T^- is unidimensional iff T is unidimensional.

In this paper, we mainly deal with the $\varnothing$-reduction property (definition 2.1). The $\varnothing$-reduction property together with the minimality condition ensures that T is not far from T^- if T is stable. But the $\varnothing$-reduction property is not so strong for an unstable theory. In fact there is a theory T such that T has the $\varnothing$-reduction property over P, is minimal over P, and the number of models of T is more than that of T^-.

EXAMPLE. Let A be a model whose theory has uncountably many countable models. Let

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