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ATOMIC DECOMPOSITION FOR SOBOLEV SPACES AND FOR THE C_p^α SPACES ON GENERAL DOMAINS

Dedicated to Professor Satoru Igari on the occasion of his sixtieth birthday

By

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§ 1. Introduction

Atomic decomposition for the Hardy spaces H^p, $0 < p \leqq 1$, is well known. In this paper, we shall give a variant of atomic decomposition which applies to the Sobolev spaces and to the C_p^α spaces of DeVore and Sharpley ([DS]) on general domains. In this section, we shall briefly review our results.

We shall first fix several notations which will be used throughout the paper. In this paper we consider functions defined on $\boldsymbol{R}^n$ or on a subset of $\boldsymbol{R}^n$; the letter n always denotes the dimension of the basic space $\boldsymbol{R}^n$. We also use the letters k, α, p, and Ω in the following fixed meaning: k denotes a nonnegative integer; α denotes a positive real number; p denotes a positive real number or ∞; and Ω denotes an open subset of $\boldsymbol{R}^n$. We shall call a Lebesgue measurable function merely a function. For a Lebesgue measurable set $E \subset \boldsymbol{R}^n$, the $L^p(E)$-quasinorm of a function f on E is defined by

$$\|f\|_{p,E} = \|f; \Gamma(1/p; E)\| = \left(\int_E |f(x)|^p dx\right)^{1/p}$$

with the usual modification in the case $p = \infty$, and the set of functions f on E such that $\|f\|_{p,E} < \infty$ is denoted by $L^p(E)$ or by $\Gamma(1/p; E)$. (Thus the two symbols $\|f\|_{p,E}$ and $\|f; \Gamma(1/p; E)\|$ denote exactly the same thing and so do the two symbols $L^p(E)$ and $\Gamma(1/p; E)$; we shall use whichever will be convenient.) We often abbreviate $\|f\|_{p,\boldsymbol{R}^n} = \|f; \Gamma(1/p; \boldsymbol{R}^n)\|$ to $\|f\|_p = \|f; \Gamma(1/p)\|$ and $L^p(\boldsymbol{R}^n) = \Gamma(1/p; \boldsymbol{R}^n)$ to $L^p = \Gamma(1/p)$. The Lebesgue measure of $E \subset \boldsymbol{R}^n$ is denoted by $|E|$. For $x = (x_1, \ldots, x_n) \in \boldsymbol{R}^n$, we write

$$|x| = \left(\sum_{j=1}^n x_j^2\right)^{1/2} \quad \text{and} \quad \|x\| = \max\{|x_i|; i = 1, \ldots, n\}.$$

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