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A RESULT EXTENDED FROM GROUPS TO HOPF ALGEBRAS

By

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We work over an algebraically closed field k of characteristic 0.

The aim of this short note is to prove the following theorem, which is an extension of a well-known fact on finite groups to finite dimensional semisimple Hopf algebras.

THEOREM. *Let p be an odd prime which is congruent to* 2 *modulo* 3. *Then a semisimple Hopf algebra of dimension* $3p$ *is isomorphic to the group-like Hopf algebra* kC_{3p} *of the cyclic group* C_{3p} *of order* $3p$.

This adds a result to the classification lists of semisimple Hopf algebras obtained recently by Larson-Radford [LR3], Zhu [Z], Masuoka [M1-3] and Fukuda [F].

First we show the following:

PROPOSITION 1. *Let p, q be primes such that $p<q$ and $q \not\equiv 1$ modulo p. Suppose that a semisimple Hopf algebra of dimension pq has a non-trivial group-like. Then A is isomorphic to kC_{pq}.*

PROOF. By the Nichols-Zoeller Theorem [NZ, Thm.7] the order of the group $G(A)$ of the group-likes in A devides the dimension $\dim A$ of A. Hence it follows by assumption that there is a Hopf subalgebra K of A isomorphic to either kC_p or kC_q. Let e_A (resp. e_K) be the primitive idempotent in A (resp. in K) sent to 1 by the counit ε_A of A (resp. ε_K of K). These idempotents e_A, e_K are contained in the character ring $C_k(A^*)$ of the dual Hopf algebra $A^* = \mathrm{Hom}_k(A, k)$, which is defined to be the subalgebra of A spanned by the characters of A^* [Z, Page 54]. Hence we have $e_K = e_A + e_2 + \cdots + e_r$, a sum of orthogonal primitive idempotents in $C_k(A^*)$. Note $\dim e_A A = 1$. Since A^* is also semisimple by [LR1, Thm. 3.3], we can apply [Z, Thm.1] to have that

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