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GLOBAL BEHAVIOURS OF CIRCLES IN A COMPLEX HYPERBOLIC SPACE

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0. Introduction

Let M be a complete Riemannian manifold. A curve γ on M parametrized by its arc length s is called a *circle* if it satisfies the following equations

$$\nabla_s X_s = kY_s, \nabla_s Y_s = -kX_s, \quad \text{and} \quad X_s = \dot{\gamma}(s)$$

for some positive constant k and a field of unit vectors Y_s along γ. Here ∇_s denotes the covariant differentiation along γ with respect to the Riemannian connection ∇ of M. The positive constant k is called the *curvature* of γ. For given a positive k and an orthonormal pair of vectors $u, v \in T_xM$ at a given point $x \in M$, we have a unique circle γ defined for $-\infty < s < \infty$ such that $\gamma(0) = x, \dot{\gamma}(0) = u$ and $(\nabla_s \dot{\gamma}(s))_{s=0} = kv$ (c.f. [7]). On a manifold of constant curvature the feature of circles with curvature k is well-known. On a Euclidean space $\boldsymbol{R}^n$ they are circles (in usual sense of Euclidean geometry) of radius $1/k$. On a sphere $S^n(c)$ of constant curvature c, they are small circles with prime period $2\pi/\sqrt{k^2+c}$. In these cases all circles are closed. Here we call a circle γ *closed* if there exists nonzero constant s_0 with $\gamma(s_0) = \gamma(0)$, $X_{s_0} = X_0$ and $Y_{s_0} = Y_0$. The minimum positive s_0 satisfying these equalities is called the prime period of γ. On a real hyperbolic space $H^n(-c)$ of constant curvature $-c$, the feature of circles is different from these two cases (c.f. [5]). When the curvature k of a circle is greater than $\sqrt{c}$ they are still closed with prime period $2\pi/\sqrt{k^2-c}$. But when $k \leq \sqrt{c}$ they are unbounded. Similarly on a Hadamard surface it is known that circles are unbounded if their curvature is smaller than the square root of the absolute value of the upper bound of the curvature of the surface (see [2]).

In this paper we study global behaviours of circles on a complex hyperbolic space $\boldsymbol{CH}_n(-c)$ of holomorphic sectional curvature $-c$. For a circle γ on a Kaehler manifold (with complex structure J and with metric $\langle\ ,\ \rangle$) we have an

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