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ISOPARAMETRIC HYPERSURFACES IN A SPACE FORM AND METRIC CONNECTIONS

Dedicated to Professor Yoshihiro Tashiro for his 70th birthday

By

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1. Introduction

As is known, homogeneous hypersurfaces in a space form are isoparametric (for instance, see [2], [4], [5], [6] and [7]), but there are many non-homogeneous isoparametric hypersurfaces ([3] and [8]). On the other hand, T. Tricerri and L. Vanhecke ([9]) have introduced the notion of a homogeneous Riemannian structure on a Riemannian manifold, following the homogeneous results which were given by W. Ambrose and I. M. Singer ([1]).

The main purpose of this paper is to study the existence of a metric connection on hypersurfaces in a space form such that this connection determines the hypersurfaces to be isoparametric, and to investigate some properties of isoparametric hypersurface with the connection. This metric connection makes the shape operator of the hypersurfaces parallel and the torsion tensor of this connection gives rise to a homogeneous Riemannian structure under a certain conditions. In fact, after a brief survey of a hypersurface in a space of constant curvature in section 2, we give the above mentioned connection (see Theorem 3.1), and investigate properties of isoparametric hypersurfaces in a space form in section 3. In section 4 we study hypersurface with a homogeneous Riemannian structure in a space of constant curvature. We show that a connection defined by the homogeneous Riemannian structure and the Levi-Civita connection makes the shape operator parallel, and some properties concerned with isoparametric ones are obtained.

2. Preliminaries

Let $(\hat{M}^{n+1}(c), \hat{g})$ be an $(n+1)$-dimensional space of constant curvature c, that is, a Riemannian manifold with the curvature form $\hat{R}$ defined by

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