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SELFINJECTIVITY OF RINGS RELATIVE TO LAMBEK TORSION THEORY

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Throughout this note R stands for an associative ring with identity, modules are unitary modules and torsion theories are Lambek torsion theories. We use the prefix "τ–" to mean "relative to Lambek torsion theory".

In this note we call a ring R left τ-selfinjective if $\mathrm{Ext}_R^1(X, R)$ is torsion for every left R-module X. Our main aim is to characterize left τ-selfinjective rings R by a certain kind of linear compactness. Recall that a module X is called absolutely pure if $\mathrm{Ext}_R^1(-, X)$ vanishes on the finitely presented modules. Also, let us call a module X semicompact if $\varprojlim \pi_\lambda$ is an epimorphism for every inverse system of epimorphisms $\{\pi_\lambda : X \to Y_\lambda\}_{\lambda\in\Lambda}$ with the Y_λ torsionless. Then, as pointed out by Stenström [18], the argument of Matlis [13, Propositions 2 and 3] yields that a ring R is left selfinjective if and only if it is left absolutely pure and right semicompact. It is shown in [9] that $\mathrm{Ext}_R^1(R/I, R)$ is torsion for every left ideal I of R if and only if R is τ-absolutely pure and right τ-semicompact. However, since τ-epimorphisms are not necessarily set-theoretic surjections, Baer's lemma does not work. Namely, even if $\mathrm{Ext}_R^1(R/I, R)$ is torsion for every left ideal I of R, R is not necessarily left τ-selfinjective. So we need a rather strong notion of linear compactness to characterize left τ-selfinjective rings R.

We are also concerned with an arbitrary class of left R-modules $\mathscr{C}$ which contains ${}_RR$ and is closed under taking factor modules and extensions. We ask when every submodule X of $E({}_RR)$, the injective envelope of ${}_RR$, with $X \in \mathscr{C}$ is torsionless. In various situations, this problem has been considered by several authors (e.g., [3], [1], [16], [20], [2], [6], [7], [4], [15] and [8]). As a particular case, we study the class of all τ-finitely generated modules.

In the following, we denote by $\mathrm{Mod}\, R$ the category of left R-modules. Right R-modules are considered as left R^{op}-modules, where R^{op} denotes the opposite ring of R. Sometimes, we use the notation ${}_RX$(resp. X_R) to stress that

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