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ON AUTOMORPHISMS OF A CHARACTER RING

Dedicated to Professor Tosiro TSUZUKU

By

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1. Introduction

Throughout this paper $G, Z(G)$ and C denote a finite group, the center of G and the field of complex numbers respectively. For a finite set S, we denote the number of elements in S by $|S|$.

Let $Irr(G)$ be the full set of irreducible C-characters of G and $X(G)$ be the character ring of G. If R is any subring of C, we write $RX(G)$ to denote the R-algebra of R-linear combinations of irreducible C-characters of G.

Suppose G and H are finite groups. Weidman showed that if $X(G)$ is isomorphic to $X(H)$, then G and H have the same character table.

In addition Saksonov proved the following theorem, which is a strengthened version of Weidman's theorem.

THEOREM 1.1. (Saksonov) *Suppose R is the ring of all algebraic integers and there exists an R-algebra isomorphism ϕ from $RX(G)$ onto $RX(H)$. If $Irr(G)=\{\chi_1,\cdots,\chi_h\}$ and $Irr(H)=\{\psi_1,\cdots,\psi_h\}$, then the following holds:*

(i) *The character tables of G and H are the same.*

(ii) $\phi(\chi_i)=\varepsilon_i\psi_{i'}$ $(i=1,\cdots,h)$ *where the ε_i are roots of unity and $i\rightarrow i'$ is a permutation.*

From now on we assume that R is the ring of all algebraic integers. Then in this paper we intend to prove the following theorem.

THEOREM 1.2. *Suppose G and H are finite groups. Then we have*

(i) *If u is a central element in G and $\tau_u: RX(G)\rightarrow RX(G)$ is the map defined by $\chi\rightarrow(\chi(u)/\chi(1))\chi$ where $\chi\in Irr(G)$ and 1 is the identity element of G, then τ_u is an R-automorphism of $RX(G)$. Furthermore the map $u\rightarrow\tau_u$ is a group isomorphism of $Z(G)$ onto a subgroup $T=\{\tau_u \mid u\in Z(G)\}$ of $Aut(RX(G))$.*

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