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ALMOST KÄHLER MANIFOLDS OF CONSTANT HOLOMORPHIC SECTIONAL CURVATURE

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1. Introduction

An almost Hermitian manifold $M=(M,J,g)$ is called an almost Kähler manifold if the corresponding Kähler form Ω is closed, or equivalently,

$$g((\nabla_X J)Y,Z)+g((\nabla_Y J)Z,X)+g((\nabla_Z J)X,Y)=0,$$

for all smooth vector fields X, Y, Z on M.

Concerning the integrability of the almost complex structure of an almost Kähler manifold, the following conjecture of S. I. Goldberg is well-known ([1]):

The almost complex structure of a compact Einstein almost Kähler manifold is integrable (and therefore the manifold is Kähler).

As concerns this conjecture, some progress has been made under some curvature conditions: K. Sekigawa proved that the above conjecture is true when the scalar curvature is non-negative [8, 9]. A complete almost Kähler manifold of constant sectional curvature is a flat Kähler manifold [4, 5, 10]. A 4-dimensional compact almost Kähler manifold which is Einstein and $*$-Einstein is a Kähler manifold [10].

In connection with these results, the author proved that a compact almost Kähler manifold of constant holomorphic sectional curvature $c\geqq 0$ which satisfies the curvature condition (b) is Kähler [7]. (The condition (b) will be given in the next section.)

In this note, we shall show that, in the case where the dimension of the manifold is four, the above statement can be improved. Namely, we shall prove that a four-dimensional compact almost Kähler manifold of constant holomorphic sectional curvature which satisfies the RK-condition is a Kähler manifold.

Throughout this paper, we assume that the manifold under consideration to be connected and of class C^∞.

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