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# ON THE EXISTENCE OF POSTPROJECTIVE COMPONENTS IN THE AUSLANDER-REITEN QUIVER OF AN ALGEBRA

By

P. DRÄXLER and J. A. de la PEÑA

Let $k$ be an algebraically closed field and $A$ be a basic finite-dimensional $k$-algebra of the form $A = kQ/I$, where $Q$ is a quiver (= finite oriented graph) and $I$ is an admissible ideal of the path algebra $kQ$, see [3]. In this work we assume that $Q$ has no oriented cycles.

Let $\mathrm{mod}_A$ denote the category of finite dimensional left $A$-modules. For each indecomposable non-projective $A$-module $X$, the Auslander-Reiten translate $\tau_A X$ is an indecomposable non-injective module. The Auslander-Reiten quiver $\Gamma_A$ has as vertices representatives of the isoclasses of the finite dimensional indecomposable $A$-modules, there are as many arrows from $X$ to $Y$ as $\dim_k \mathrm{rad}_A(X,Y)/\mathrm{rad}_A^2(X,Y)$. In this paper we do not distinguish between a module and its corresponding isoclass. A connected component $\mathcal{P}$ of $\Gamma_A$ is *postprojective* if $\mathcal{P}$ has no oriented cycles and each module $X$ in $\mathcal{P}$ has only finitely many predecessors in the path order of $\mathcal{P}$. Several important classes of algebras have postprojective components: hereditary algebras [3, 6], algebras satisfying the separation condition [1, 2], tilted algebras [8].

The aim of this work is to find necessary and sufficient conditions for the existence of postprojective components in $\Gamma_A$. In section 1 we give an algorithmic procedure to decide the existence of postprojective components. In section 2 we consider a one-point extension algebra $A = B[M]$ such that all indecomposable direct summands of $M$ belong to postprojective components of $\Gamma_B$, then we give conditions that assure that the projective $A$-module $P$ with rad $P = M$ lies in a postprojective component of $\Gamma_A$. In section 3 we consider some special cases. We recall that once identified a postprojective component $\mathcal{P}$ of $\Gamma_A$, the modules on $\mathcal{P}$ may be constructed using the *knitting procedure* [3]. In [5], an algorithmic procedure which makes essential use of the knitting procedure is given to construct all the postprojective components of $\Gamma_A$.

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