

THE ω -CONSISTENCY OF ELEMENTARY ANALYSIS

By

Kokio SHIRAI

§0. Introduction

Let EA be the formal system of elementary analysis. The following is proved by Schütte [7] that we have a proof of the consistency of EA by transfinite induction up to $\varepsilon_{\varepsilon_0}$.

The purpose of this paper is to prove the following two theorems:

THEOREM 1. *The ω -consistency of EA can be proved by applying transfinite induction up to $\varepsilon_{\varepsilon_1}$ for an elementary number theoretical proposition, together with exclusively elementary number theoretical techniques.*

THEOREM 2. *The ω -consistency of EA cannot be proved by applying transfinite induction to numbers below $\varepsilon_{\varepsilon_1}$ for the elementary number theoretical propositions, together with exclusively elementary number theoretical techniques.*

The proof of theorem 1 is carried out as the same line as that of ω -consistency of elementary number theory EN by Hanatani [6], which is an application of Gentzen's consistency proof of EN [3].

The greater part of technical terms and conventions are adopted from English translation [8] of the works of G. Gentzen and Takeuti [9] and Schütte [7].

The author would like to express his hearty gratitude to the late Prof. Shôji Maehara and Prof. Mitio Takano who encouraged him with many valuable advices.

§1. Formal systems EA and EA(M)

We define the formal systems EA and EA(M).

1.1. As primitive symbols we use

1. Denumerably infinitely many free and bound number variables.

Received July 4, 1994

Revised November 7, 1994