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CONSTANT SCALAR CURVATURES ON WARPED PRODUCT MANIFOLDS

By

"Dedicated to Professor Tsunero Takahashi on his 60th birthday"

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1. Introduction

In a recent study [D.D.], F. Dobarro and E. L. Dozo have studied from the viewpoint of partial differential equations and variational methods, the problem of showing when a Riemannian metric of constant scalar curvature can be produced on a product manfild $B\times F$ by a warped product construction applied to the two Riemannian manifolds (B,g_B) and (F,g_F), especially in the case when the fibre (F,g_F) is of constant curvature. Particularly, in Theorem 3.6 of [D.D.], the uniqueness of the warping function is considered. In [D.D.], the eigenvalue problem for the elliptic operator $Lu=-\frac{4n}{n+1}\Delta u+Ru$ of a warped product $B\times_f F$ of Riemannian manifolds B and F, where Δ is the Laplacian on B and R is the scalar curvature on B, is studied. Basically, the fact that the operator $L-\lambda I: C^{2,\alpha}(M)\to C^{\alpha}(M)$ is an isomorphism for some λ, is employed.

For Riemannian manifolds, warped products have been useful in producing examples of spectral behavior, examples of manifolds of negative curvature (cf. [B.O.], [D.G.], [D.D.], [Eb], [Ej], [K.K.P.], [M.M.]), and also in studying L_2-cohomology (cf. [Z.]).

Perhaps even more interestingly on physical grounds than purely Riemannian constructions employing warped products, many of the known exact solutions of the Einstein field equations of General Relativity are warped product metrics of the form $B\times_f F$ where (B,g_B) is a Lorentzian manifold and (F,g_F) is a Riemannian manifold. A most notable class of examples are the Robertson-

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