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UNIMODULARITY OF FINITE DIMENSIONAL HOPF ALGEBRAS

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Introduction.

D. Radford [4] proved that a finite dimensional Hopf algebra A is a symmetric algebra if and only if

(1) A is unimodular, and

(2) the square of the antipode s^2 is an inner algebra-automorphism.

It is well-known that the 4-dimensional Hopf algebra of Sweedler shows that Condition (2) does not necessarily imply Condition (1).

We present in this paper an example which shows that the converse does not hold either.

The construction.

Let n be a positive integer and m_i an integer $\geqq 2$ for $1\leqq i\leqq n$. Let k be a field that contains a primitive m_ith root of unity η_i and let $\omega\in k$ be an element satisfying $\omega^{m_i}=1$. We may assume that ω is a primitive mth root of unity for some positive integer m. We note that m divides m_i.

As a general case we shall construct a Hopf algebra B over k, which is generated as an algebra by g_i, x_i subject to the relations;

$$g_jg_i=g_ig_j,\quad g_k^{m_k}=1,\quad x_k^{m_k}=0,\quad x_jg_i=\omega g_ix_j,$$
$$x_kg_k=\eta_kg_kx_k,\quad x_ig_j=\omega^{-1}g_jx_i,\quad x_jx_i=\omega x_ix_j,\quad \text{for } 1\leqq k\leqq n,\quad 1\leqq i\lneqq j\leqq n.$$

First, let $F=k[G_1,\cdots,G_n,G_1^{-1},\cdots,G_n^{-1},X_1,\cdots,X_n]$ be the free algebra on $3n$ noncommuting indetarminates. We form the so-called free Hopf algebra

$$F=k[G_1,\cdots,G_n,G_1^{-1},\cdots,G_n^{-1},X_1,\cdots,X_n]/(G_iG_i^{-1}-1,G_i^{-1}G_i-1).$$

The coalgebra structure maps $F\to F\otimes F$ and $F\to k$ are the algebra-homomorphisms determined by

$$G_i\mapsto G_i\otimes G_i,\quad X_i\mapsto X_i\otimes G_i+1\otimes X_i,$$

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