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A CHARACTERIZATION OF PSEUDO-EINSTEIN REAL HYPERSURFACES IN A QUATERNIONIC PROJECTIVE SPACE

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0. Introduction.

Let $\boldsymbol{HP}^n$ be a quaternionic projective space, $n \geqq 3$, with metric G of constant quaternionic sectional curvature 4, and let M be a connected real hypersurface of $\boldsymbol{HP}^n$. Let ξ be a unit local normal vector field on M and $\{I, J, K\}$ a local basis of the quaternionic structure of $\boldsymbol{HP}^n$ (cf. [4]). Then $U_1 = -I\xi, U_2 = -J\xi, U_3 = -K\xi$ are unit vector fields tangent to M. We call them *structure vectors*. Now we put $f_i(X) = g(X, U_i)$, for arbitrary $X \in TM$, $i = 1, 2, 3$, where TM is the tangent bundle of M and g denotes the Riemannian metric induced from the metric G. We denote D and $D^{\perp}$ the subbundles of TM generated by vectors perpendicular to structure vectors, and structure vectors, respectively. There are many theorems from the point of view of the second fundamental tensor A of M (cf. [1], [8] and [9]). It is known that if M satisfies $g(AD, D^{\perp}) = 0$ then there is a local basis of quaternionic structure such that structure vectors are principal vectors. Berndt classified the real hypersurfaces which satisfy this condition (cf. [1]). On the other hand we know some results on real hypersurfaces of $\boldsymbol{HP}^n$ in terms of the Ricci tensor S of M (cf. [3] and [8]). If the Ricci tensor satisfies that $SX = aX + b\sum_{i=1}^{3} f_i(X)U_i$ for some smooth functions a and b on M, then M is called a pseudo-Einstein real hypersurface of $\boldsymbol{HP}^n$. This notion comes from the problem for the real hypersurfaces in complex projective space $\boldsymbol{CP}^n$. Kon studied it under the assumption that they have constant coefficients (cf. [5]) and Cecil and Ryan gave a complete classification (cf. [2]). In [8] Martinez and Perez studied pseudo-Einstein real hypersurfaces of $\boldsymbol{HP}^n$, $n \geq 3$ under the condition that a and b are constant. Using Berndt's classification we show that we do not need the assumption. The main purpose of this paper is to provide a characterization of pseudo-Einstein real hypersurface in $\boldsymbol{HP}^n$ by using an estimate of the length of the Ricci tensor S, which is a quaternionic version of a result of Kimura and

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