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ON ISOMORPHISMS OF A BRAUER CHARACTER RING ONTO ANOTHER

Dedicated to Professor Hiroyuki Tachikawa

By

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1. Introduction

Throughout this paper G, Z and Q denote a finite group, the ring of rational integers and the rational field respectively. Moreover we write $\overline{Z}$ to denote the ring of all algebraic integers in the complex numbers and $\overline{Q}$ to denote the algebraic closure of Q in the field of complex numbers. For a finite set S, we denote by $|S|$ the number of elements in S.

Let $Irr(G)=\{\chi_1,\cdots,\chi_h\}$ be the complete set of absolutely irreducible complex characters of G. Then we can view $\chi_1,\cdots,\chi_h$ as functions from G into the complex numbers. We write $\overline{Z}R(G)$ to denote the $\overline{Z}$-algebra spanned by $\chi_1,\cdots,\chi_h$. For two finite groups G and H, let λ be a $\overline{Z}$-algebra isomorphism of $\overline{Z}R(G)$ onto $\overline{Z}R(H)$. Then we can write

$$\lambda(\chi_i)=\sum_{j=1}^{h} a_{ij}\chi'_j, \quad (i=1,\cdots,h)$$

where $a_{ij}\in\overline{Z}$ and $Irr(H)=\{\chi'_1,\cdots,\chi'_h\}$. In this case we write A to denote the $h\times h$ matrix with (i,j)-entry equal to a_{ij} and say that A is afforded by λ with respect to $Irr(G)$ and $Irr(H)$.

As is well known, concerning the isomorphism λ, we have the following two results, which seem to be most important. (For example see Theorem 1.3 (ii) and Lemma 3.1 in [5])

(i) $|c_G(c_i)|=|c_H(c'_{i'})|$, $(i=1,\cdots,h)$ *where* $\{c_1,\cdots,c_h\}$ *and* $\{c'_{1'},\cdots,c'_{h'}\}$ *are complete sets of representatives of the conjugate classes in* G *and* H *respectively and* $c_i\xrightarrow{\lambda}c'_{i'}$ $(i=1,\cdots,h)$. (Concerning a symbol "$c_i\xrightarrow{\lambda}c'_{i'}$", see the definition in [5] and also the definition in section 2 in this paper)

(ii) *A is unitary where A is the matrix afforded by λ with respect to $Irr(G)$ and $Irr(H)$.*

In this paper our main objective is to give a necessary and sufficient condition

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