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THE STABLE TOPOLOGY OF MODULI SPACES OF PERIODIC INSTANTONS

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1. Introduction and statement of the result

Let M be a smooth 4-manifold which admits an open subset K with one end N and an open submanifold W_0 with two ends N_-, N_+. $W_1, W_2, \cdots$ denote copies of W_0. The 4-manifold M will be called end-periodic if it admits a decomposition $M = K \cup_N W_0 \cup_N W_1 \cup \cdots$, where $N \subset K$ is identified with the end N_- of W_0 and the end N_+ of W_0 is identified with the end N_- of W_1 and so on. Let Y be the compact oriented 4-manifold which is obtained from W_0 by identifying the two ends. The manifold Y has a Z-cover $\tilde{Y} = \cdots_N W_{-1} \cup_N W_0 \cup_N W_1 \cdots$ with projection $\pi : \tilde{Y} \to Y$. A geometric object on M, a vector bundle, a connection, a differential operator, a Riemannian metric etc. will be called end-periodic if its restriction on End$M$= $W_0 \cup_N W_1 \cdots$ is the pull back by π of an object on Y. By making choose a smooth function $s: W_0 \to [0,1]$ such that $s|N_- = 0$ and $s|N_+ = 1$, we obtain a smooth step function t on M such that $t(x) = n + s(x)$ if $x \in W_n$.

Let $P \to M$ be an end-periodic principal $SU(2)$-bundle, and A_0 be an end-periodic connection on P which is gauge equivalent over End$M$ to the product connection on End$M \times SU(2)$. Then by the lemma 7.1 in [7]

$$l = (1/8\pi^2)\int_M tr(F_{A_0} \wedge F_{A_0})$$

is an integer, where tr( ) is the trace on the adjoint representation of the group $SU(2)$. Let $E \to M$ be an end-periodic vector bundle which is associated to the principal bundle $P \to M$. Put $L^2{}_{\mathrm{loc}}(E) = \{\text{section } u; u \in L^2(E|_A)$ for every measurable $A \subset\subset M\}$, where we assume that the set A has a finite measure, and denote by $\|\cdot\|_{A_0}$ the norm by the covariant derivative $\nabla_{A_0} : C_0^\infty(E) \to C_0^\infty(E \otimes T^*M)$ of compactly supported smooth sections, further $\nabla_{A_0}^{(j)}$ denotes the j-times iterated derivative $\nabla_{A_0} \cdots \nabla_{A_0}$. For $\delta > 0$, put

$$\mathscr{A}_k(\delta) = \{A_0 + a; a \in L^2{}_{5,\mathrm{loc}}(adP \otimes T^*M) \text{ with norm } \|a\|_{A_0} < \infty\},$$

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