TSUKUBA J. MATH.
Vol. 20 No. 1 (1996), 181–190

TOTALLY GEODESIC SUBMANIFOLDS OF NATURALLY REDUCTIVE HOMOGENEOUS SPACES

By

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1. Introduction.

B. Y. Chen and T. Nagano [2] investigated the totally geodesic submanifolds in Riemannian symmetric spaces, and as one of their results, the following holds.

FACT 1.1. *Spheres and hyperbolic spaces are the only simply connected, irreducible symmetric spaces admitting a totally geodesic hypersurface.*

In this paper we shall study totally geodesic submanifolds in the naturally reductive homogeneous spaces which are known as a natural generalization of Riemannian symmetric spaces.

At first, in a naturally reductive space (M, g), we express a necessary and sufficient condition of the existence of a totally geodesic submanifolds in the language of the Lie algebra of a Lie group of isometries of M (Theorem 3.2), which generalizes the notion of the Lie triple system due to E. Cartan.

Next, as an application of that, by making use of the results in Kowalski and Vanhecke [5, 6], we shall prove that simply connected, irreducible naturally reductive spaces of dimension n ($n = 3, 4, 5$) admitting a totally geodesic hypersurface are spheres and hyperbolic spaces (Theorem 4.1).

The author would like to thank R. Takagi and K. Tsukada for their kind suggestions.

2. Totally geodesic submanifolds of Riemannian spaces.

Let (M, g) be a Riemannian manifold and ∇ the Levi-Civita connection of (M, g). Let p be a point of M and u a vector in the tangent space T_pM to M at p. P_u denotes the parallel transport with respect to ∇ along the geodesic $\gamma_u(t) = \mathrm{Exp}_p(tu)$ from p to $\gamma_u(1)$, where Exp denotes the Riemannian exponential map. Let R be the curvature tensor defined by

$$R(X,Y) = \nabla_{[X,Y]} - [\nabla_X, \nabla_Y],$$

Received May 6, 1994