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THE MODULI SPACE OF ANTI-SELF-DUAL CONNECTIONS OVER HERMITIAN SURFACES

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1 Introduction

Let (M, g) be a compact Hermitian surface with an orintation induced by the complex structure of M, and P a principal bundle over M with structure group $SU(n)$. Then a canonical representation ρ of $SU(n)$ induces a smooth complex vector bundle $E=P\times_{\rho}\mathbf{C}^{n}$. A necessary and sufficient condition for a $SU(n)$-connection D on E to be an anti-self-dual connection is that the curvature of D is a differential 2-form of type (1,1), and is orthogonal to the fundamental form Φ of (M, g). Hence, a holomorphic structure is induced on E and hence on $\mathrm{End}^{0}E$ (the subbundle of $\mathrm{End}E$ consisting of endomorphisms with trace 0) by an anti-self-dual connection D. Itoh ([4]) showed that the moduli space of anti-self-dual connections over Kähler surfaces is a complex manifold. We will extend this result over Kähler surfaces to over Hermitian surfaces, which are not necessarily Kählerian.

Let K_{M} be a canonical line bundle over M. We define $\tilde{H}_{D}=H_{D}^{0}(M;\mathcal{O}(\mathrm{End}^{0}E\otimes K_{M})$ as the space of holomorphic sections, where $\mathrm{End}^{0}E$ is endowed with the holomorphic structure induced from the irreducible anti-self-dual connection D. We denote by $\mathcal{M}$ the moduli space of irreducible anti-self-dual connections (the quotient space of irreducible anti-self-dual connections by the gauge transformation group $SU(E)$), and set $\mathcal{M}_{0}$ as follows: $\mathcal{M}_{0}=\{[D]\in\mathcal{M}\,\tilde{H}_{D}=(0)\}$. Then we obtain the following

THEOREM 1. *Let M be a compact Hermitian surface. If $\mathcal{M}_{0}$ is not empty, then $\mathcal{M}_{0}$ is a complex manifold.*

We can make H_{D} vanish under a certain condition. On a Hermitian manifold

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