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ON A REFINEMENT OF ANTI-SOUSLIN TREE PROPERTY

To the memory of the late Dr Simauti Takakazu,
the late Dr Maehara Shoji and the late Dr Hirose Ken

By

Masazumi HANAZAWA

§0. Introduction.

The field of uncountable trees contains so many unusual parts that has provided various examples and counter-examples in the fields around set theory and general topology (see Todorcevic [5]). A Souslin tree and a special Aronszajn tree are famous examples among them. The former is characterized mainly by the property that it has no uncountable antichain, and the latter by the one that it is a countable union of antichains. As seen here, the antichain properties often play a main role in describing tree characters. Anti-Souslin tree with which we shall concern is also in such a case. It is defined as a tree in which every uncountable set contains an uncountable antichain (Baumgartner [1], See Remark 1). In the above, countability and uncountability are the only scales for the size of infinite antichains. More refined scales of meaningful sense appears in Devlin and Shelah [3] and Shelah [4], where they introduce the notions of "stationary" and "club" for the subsets of a tree, and prove that e.g. for an ω_1-tree:

(*) it is collectionwise hausdorff under interval topology, if it has no stationary antichain,

(**) the existence of stationary antichains does not imply the existence of club antichains.

Thus it is expected that there would be some significant differences between those notions that are obtained from the definition of anti-Souslin property by replacing one or both occurrences of the word "uncountable" by "stationary" or "club". The present paper investigates the implicational relationships between these new notions. Consequently they are reduced to four different notions. We also try to

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