Sp(n)-EQUIVARIANT HARMONIC MAPS BETWEEN COMPLEX PROJECTIVE SPACES

Dedicated to Professor Hideki Ozeki on his sixtieth birthday

By

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Introduction

On existence of harmonic maps, Guest [2] constructed equivariant harmonic maps from a flag manifold to a complex Grassmannian manifold, and Ohnita [5] developed a method of studying equivariant maps from a compact homogeneous space to a complex projective space and investigated equivariant harmonic maps from a compact irreducible Hermitian symmetric space to a complex projective space, in detail. In particular, Ohnita classified equivariant harmonic maps relative to a unitary group between complex projective spaces.

In this paper, we study existence and harmonicity of $Sp(n)$-equivariant maps between complex projective spaces, by using the fact the symplectic group $Sp(n)$ acts a $(2n - 1)$-dimensional complex projective space CP^{2n-1} transitively. In section 4 we determine all complex irreducible representations of $Sp(n)$, which define $Sp(n)$-equivariant maps from CP^{2n-1} to CP^m (Theorem 4.3), with the aid of the restriction rule of representations of $Sp(n)$, due to Koike and Terada [3, 4], Zhelobenko [6]. In section 5 we prove that the associated $Sp(n)$-equivariant maps are harmonic for any $Sp(n)$-invariant Riemannian metric on CP^{2n-1} (Theorem 5.2). In particular, we get $Sp(n)$-equivariant minimal immersions from CP^{2n-1} to CP^m, but not $SU(2n)$-equivariant.