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EXISTENCE AND ASYMPTOTIC BEHAVIOR OF WEAK SOLUTIONS TO SEMILINEAR HYPERBOLIC SYSTEMS WITH DAMPING TERMS

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1. Introduction

Let Ω be a bounded domain of $\boldsymbol{R}^k$ with Lipschitz boundary $\partial\Omega$. We consider the following system of hyperbolic equations for a map $u:\Omega\times(0,\infty)\to\boldsymbol{R}^\ell$:

$$\begin{aligned}(1.1)\qquad & a_{ij}(x)D_t^2u^i(x,t)-D_\beta(b_{ij}^{\alpha\beta}(x)D_\alpha u^i(x,t))+c_{ij}(x)\|u(x,t)\|_c^{m-2}u^i(x,t)\\ &+a_{ij}(x)D_tu^i(x,t)=0 \ \text{ in } \Omega\times(0,\infty),\ \ j=1,\dots,\ell,\end{aligned}$$

where $D_t=\partial/\partial t, D_\alpha=\partial/\partial x^\alpha, \|u(x,t)\|_c=(c_{ij}(x)u^i(x,t)u^j(x,t))^{1/2}$ and $m>1$. Here and in the following, summation over repeated indices is understood, the greek indices run from 1 to k, and the latin ones from 1 to ℓ. We assume that the coefficients $a_{ij}(x)$, $b_{ij}^{\alpha\beta}(x)$ and $c_{ij}(x)$ are bounded functions defined on Ω and satisfy the conditions

$$(1.2)\qquad \begin{cases} a_{ij}(x)\xi^i\xi^j\ge\lambda_0|\xi|^2 & \text{for all } \xi\in\boldsymbol{R}^\ell,\\ b_{ij}^{\alpha\beta}(x)\eta_\alpha^i\eta_\beta^j\ge\lambda_1|\eta|^2 & \text{for all } \eta\in\boldsymbol{R}^{k\ell},\\ c_{ij}(x)\xi^i\xi^j\ge\lambda_2|\xi|^2 & \text{for all } \xi\in\boldsymbol{R}^\ell,\end{cases}$$

$$(1.3)\qquad a_{ij}(x)=a_{ji}(x),\ \ b_{ij}^{\alpha\beta}(x)=b_{ji}^{\alpha\beta}(x),\ \ c_{ij}(x)=c_{ji}(x),$$

for some positive constants λ_0,λ_1 and λ_2. The initial and boundary conditions are

$$(1.4)\qquad u(x,0)=u_0(x),\ \ D_tu(x,0)=v_0(x) \ \text{ in } \Omega,$$

$$(1.5)\qquad u(x,t)=w(x) \ \text{ on } \partial\Omega\times(0,\infty),$$

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