TSUKUBA J. MATH.
Vol. 20 No. 1 (1996), 33–44

ON MINIMAL SURFACES WITH CONSTANT KÄHLER ANGLE IN CP^3 AND CP^4

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0. Introduction.

Let M be a 2-dimensional Riemannian manifold isometrically immersed in a Kähler manifold N with the complex structure J, and let $\{e_1, e_2\}$ be a local orthonormal frame on M. The Kähler angle α of M is defined to be the angle between Je_1 and e_2. The surface M is holomorphic, anti-holomorphic or totally real if and only if $\alpha = 0$, π or $\pi/2$, respectively. In [6] Chern and Wolfson pointed out that the Kähler angle plays an important role in the study of minimal surfaces in Kähler manifolds.

Here we consider the problem to classify minimal surfaces with constant Kähler angle in CP^n, where CP^n denotes the complex projective space of constant holomorphic sectional curvature 4. Concerning this problem, several results are known (see [1], [10], [8], [4] and [9]). In particular, we recall the following three facts. (I) A minimal surface with constant Kähler angle in CP^2 is either holomorphic, anti-holomorphic or totally real (see [6, (2.32)] and [8, Lemma 2.1]). (II) A pseudo-holomorphic minimal surface with constant Kähler angle in CP^3 is either holomorphic, anti-holomorphic, totally real or of constant curvature (see the proof of Theorem 9.1 of [1]). (III) A minimal 2-sphere with constant Kähler angle in CP^4 is either holomorphic, anti-holomorphic, totally real or of constant curvature (see [1, Theorem 9.1], cf. [8, p. 372]).

REMARK 1. (i) A minimal surface in CP^n is called pseudo-holomorphic if its harmonic sequence terminates at each end (see [3] and [5]).

(ii) Minimal surfaces with constant curvature and Kähler angle in CP^n are classified in [10].

In this paper we prove the following:

THEOREM 1. *Let M be a superconformal minimal surface with constant Kähler angle in CP^3. Then M is totally real.*

Received January 27, 1994. Revised June 6, 1994.