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ON EMBEDDINGS OF PERFECT GO-SPACES INTO PERFECT LOTS

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§1. Introduction

A *linearly ordered topological space* (abbreviated *LOTS*) is a triple $\langle X,\lambda,\leq\rangle$, where $\langle X,\leq\rangle$ is a linearly ordered set and λ is the usual interval topology defined by $\leq$. Throughout this paper, λ, $\lambda(\leq)$ or λ_X denote the usual interval topology on a linearly ordered set $\langle X,\leq\rangle$.

A *generalized ordered space* (abbreviated *GO-space*) is a triple $\langle X,\tau,\leq\rangle$, where $\langle X,\leq\rangle$ is a linearly ordered set and τ is a topology on X such that $\lambda\subset\tau$ and τ has a base of open sets each of which is order-convex, where a subset A of X is called *order-convex* if $x\in A$ for every x lying between two points of A. For a GO-space $\langle X,\tau,\leq\rangle$ and $Y\subset X$, $\tau|Y$ denotes the subspace topology $\{U\cap Y: U\in\tau\}$ on Y and $\leq|Y$ denotes the restricted ordering of $\leq$ on Y. If it will cause no confusion, we shall omit λ (or τ) and $\leq$, and say simply "X is a LOTS (GO-space)". A topological space $\langle X,\tau\rangle$, where τ is a topology on a set X, is said to be *orderable* if $\langle X,\tau,\leq\rangle$ is a LOTS for some linear ordering $\leq$ on X. Similarly, we say simply "X is an orderable space" if it will cause no confusion. A LOTS $Z=\langle Z,\lambda,\leq_Z\rangle$ is said to be a *linearly ordered extension* of a GO-space $X=\langle X,\tau,\leq_X\rangle$ if $X\subset Z$, $\tau=\lambda|X$ and $\leq_X=\leq_Z|X$. Furthermore, if X is closed (resp., dense) in the space $\langle Z,\lambda\rangle$, then Z is said to be a *linearly ordered c-extension* (resp., *d-extension*) of X. Similarly, an orderable space $Z=\langle Z,\tau_Z\rangle$ is said to be an *orderable c-* (resp., *d-*)*extension* of a GO-space $X=\langle X,\tau_X\leq\rangle$ if X is a closed (resp., dense) subset of Z and $\tau_X=\tau_Z|X$. Note that every GO-space has a compact linearly ordered d-extension ([5, (2.9)]).

Throughout this paper, we use the following notation: Let $\langle Y,\lambda,\leq\rangle$ be a LOTS. For a GO-space $\langle X,\tau,\leq\rangle$ with the same underlying set Y and the same order $\leq$, we write $X=GO_Y(R,E,I,L)$, where $I=\{x\in X:\{x\}\in\tau-\lambda\}$, $R=\{x\in X:[x,\rightarrow)\in\tau-\lambda\}-I$, $L=\{x\in X:(\leftarrow,x]\in\tau-\lambda\}-I$ and $E=X-(I\cup R\cup L)$.

The following problem naturally arises.

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